

Scuola Superiore di Catania

**CORSO SPECIALISTICO
Ambito giuridico-economico
a.a. 2016-2017**

Individual Choice Theory and Rationality: Axiomatic Foundations in Economics, Mathematics, and Computational Logic

• **Prerequisites.** Basic understanding of elementary set theory and propositional logic suffices. Some knowledge of microeconomics, macroeconomics, topology, combinatorics, and first-order logic may provide a better insight into some topics, but it is not strictly necessary.

• **Goal.** The objective of this course is to provide students with a broad view of the subject of *rational choice* from several perspectives: economics, mathematics, and computational logic. The analysis is structured in a large number of topics, some basic and some others at the very frontier of research. All basic topics will be covered in detail, whereas some very recent developments of the theory will only be sketched. In this way, students should be exposed to a large variety of research topics, and they can select the appealing one(s) for further study. Instructors will focus their attention on those topics on which students appear to be more responsive.

• **Structure.** The course is composed of three modules (for a total of 34 hours):

1. **Epistemological Foundations of Rationality in Economics**
2. **Mathematical Foundations of Rationality in Choice Theory**
3. **Decision Procedures and Model Theory for Rational Choices**

The content of each module is described below. There is an unusually large amount of references. They are included to either provide the basic setting of the theory or suggest further readings on the topics discussed in class. All references are listed at the end of this document, and they are labeled as either "B" (books), or "P" (papers), or "W" (work in progress).

• **Final Exam.** At the end of the course, each student will be tested on a written exam of one hour, with the goal of verifying his understanding of the main topics.

1. Epistemological Foundations of Rationality in Economics

The aim of this module is to provide students with a methodological perspective on the concept of rationality in economics. This spreads over a wide context, ranging from epistemological roots to mathematical tools.

1.1 Introduction [B5] Cognitive design of the problem: the extent of the topic and the attributions of students' learning.

1.2 Philosophical Contributions to the Concept of Rationality [W1] A brief overview of some philosophical approaches to the idea of rationality, codifying the abstract concept into concrete notions.

1.3 Rationality in Economics [B8] A methodological review of the main contributions of economists to the idea of rationality.

1.4 Modeling Rational Economic Choices [P5] An alternative model of rational choices. Applicability of the concept of rationality in economics.

1.5 Collective Choices in Modern Society [B4] Paradoxes and violation of the idea of rationality: context, framing, information. Rationality as an average field between individual and collective spaces.

2. Mathematical Foundations of Rationality in Choice Theory

The aim of this module is to provide students with a rigorous treatment of individual choice theory and the associated theory of revealed preferences. In particular, we extensively deal with the topic of rationality for the case of deterministic choices. Several very recent developments of the theory are sketched.

2.1 Individual Choice and Revealed Preference Theory [B1, B6] Choice correspondences. Revealed preference theory. Rationalizability. Axioms of choice consistency.

2.2 Classical Preference Structures and Utility Representations [B2] Binary preferences. Utility representations. Total preorders. Debreu separability.

2.3 Special Mono-Preference Structures [B1, P4] Preorders and multi-utility representations. Interval orders and semiorders. Universal semiorders.

2.4 (m, n) -Rationalizable Choices [P1, P3] Weak and strict (m, n) -Ferrers properties. (m, n) -rationalizable choices. Moneypump phenomena. Axioms of (m, n) -replacement consistency.

2.5 Multi-Rationalizability and Multi-Preferences [P2, W6] Motivation. Free and monotonic multi-rationalization. Bi-preferences. Necessary and possible preferences. Modal utility representations.

2.6 The Structure of Choices I: Revealed Indiscernibility [W2] Abstract motivation. Revealed indiscernibility. Congruency. Derived choice on the quotient set. Stabilization. Preservation of rationalizability.

2.7 The Structure of Choices II: Factorization and Shrinkability [W3] Factorization of choices. Decomposable choices. Shrinkability and bi-shrinkability. Preservation of rationalizability.

3. Decision Procedures and Model Theory for Rational Choices

The aim of this module is to provide students with basic notions and tools for the automation of reasoning in the context of rational choice theory.

3.1 Introduction to Propositional Logic [B3] Syntax. Semantics. Normal forms. The Davis-Putnam propositional decision algorithm.

3.2 The Decision Problem for the Unquantified Boolean Theory of Sets [B6] Syntax. Semantics. Canonical 'flat' form. A decision procedure based on Venn diagrams.

3.3 Unquantified Boolean Theory of Sets with Choice (UBTSC) [W4] Syntax. Semantics. Decision procedures for the theory UBTSC under axioms of choice consistency:

3.3.1 The case of standard contraction consistency;

3.3.2 The case of transitive rationalizability.

3.4 A Model-Theoretic Approach to Choice Indiscernibility [W5]

Syntax and semantics of first-order logic. Model-theoretic characterizations of indiscernibility.

References

- B1 Aleskerov, F., D. Bouyssou, and B. Monjardet, B. (2007). *Utility Maximization, Choice, and Preference* (2nd Ed.). Springer, Berlin.
- B2 Bridges, D., and G. B. Mehta (1995). *Representations of Preference Orderings*. Lecture Notes in Economics and Mathematical Systems 422, Springer, Berlin.
- B3 Fitting, M. (1996). *First-order Logic and Automated Theorem Proving* (2nd Ed.). Springer-Verlag, New York.
- B4 Nida-Rümelin, J. (2013). *Economic Rationality and Practical Reason*. Vol. 24. Springer Science & Business Media.
- B5 Rubinstein, A. (1998). *Modeling Bounded Rationality*. MIT press.
- B6 Schwartz, J. T., D. Cantone, D., and E. G. Omodeo (2011). *Computational Logic and Set Theory: Applying Formalized Logic to Analysis*. Springer-Verlag, Berlin.
- B7 Suzumura, K. (1983). *Rational Choice, Collective Decisions, and Social Welfare*. Cambridge University Press, Cambridge.
- B8 Zouboulakis, M. (2014). *The Varieties of Economic Rationality: from Adam Smith to Contemporary Behavioural and Evolutionary Economics*. Routledge.
- P1 Cantone, D., A. Giarlotta, S. Greco, and S. Watson (2016). (m, n) -rationalizable choices. *Journal of Mathematical Psychology*, forthcoming.
- P2 Giarlotta, A., and S. Greco (2013). Necessary and possible preference structures. *Journal of Mathematical Economics* 42(1): 163–172.
- P3 Giarlotta, A., and S. Watson (2014). The pseudo-transitivity of preference relations: strict and weak (m, n) -Ferrers properties. *Journal of Mathematical Psychology* 58: 45–54.
- P4 Giarlotta, A., and S. Watson (2016). Universal semiorders. *Journal of Mathematical Psychology*, forthcoming.
- P5 Michael, R. T., and G. S. Becker (1973). On the new theory of consumer behavior. *The Swedish Journal of Economics*: 378–396.
- W1 Biondo A. E., and A. Rapisarda. *Ratio Oeconomica*.
- W2 Cantone, D., A. Giarlotta, and S. Watson (submitted). Revealed indiscernibility.
- W3 Cantone, D., A. Giarlotta, and S. Watson. The structure of choices: factorizations.
- W4 Cantone, D., A. Giarlotta, and S. Watson. Decision procedures for fragments of set theory with a choice function.
- W5 Cantone, D., A. Giarlotta, and S. Watson. A model-theoretic approach to choice indiscernibility.
- W6 Giarlotta, A., and S. Watson. Multi-rationalizability.